Some simple considerations

1. Taking

$$\left[x_+ E_+, \frac{k_\perp^2}{x_+ E_+}, \mathbf{k}_\perp\right] \tag{1}$$

The maximum in light-cone variables is when $k_\perp = x_+ E_+$, i.e.

$$k_- = \frac{k_\perp^2}{x_+ E_+} = x_+ E_+ \tag{2}$$

This means that $k_+ = k_-$ or

$$k_- = \omega - k_l = \omega + k_l = k_+ \qquad \Longleftrightarrow \qquad k_l = 0 \tag{3}$$

In this case $\omega = k_\perp$ and

$$\omega = k_\perp = x_+ E_+ \tag{4}$$

should be the correct limit — i.e. without the factor of 2.

2. In the opposite limit, when $k_\perp = 0$

$$k_l = \omega \qquad \Longleftrightarrow \qquad k_+ = 2\omega; \quad k_- = 0 \tag{5}$$

and the factor of two appears

3. In general, the limit is an interpolation between the two as given in the notes by Uli Heinz (but, of course, without the substitution $E_+ = 2E$)

Given the fact that in the eikonal limit $k_\perp \ll \omega$ the two sets of variables give exactly the same result, one needs to make a choice for extending the formalism to the non-eikonal limit. There are two possible choices:

1. Write first everything in terms of the Minkovski variables

$$x_+ \to x_E; \quad E_+ \to 2E; \quad L_+ \to 2L \tag{6}$$

   and then apply the on-shell condition $k_\perp \leq \omega = xE$ — this is what is done in ASW

2. Compute everything in the light-cone variables with the appropriate integration limits.

The implementation in WHDG takes only the change $E_+ \to 2E$ [1] and ignores $x_+ \to x_E$. The question is how this change is implemented in the final calculation — at the end only the energy loss matters and not the light-cone energy loss. I.e. if the energy loss of the traversing partons is taken to be $\Delta E = x_+ E$ then the integrals are taking values where the gluon is not on-shell, as, in this case, one is taken $x_E = x_+$ and the limit $k_\perp^{\max} = 2x_+(1-x_+)E$ violates the on-shell condition for $x_+ < 1/2$.

So, the first choice above looks simpler and more natural.

[1] In fact it's not clear in the writeup how the change from L_+ to L is performed